

Total Quantum Zeno Effect beyond Zeno Time

D. Mundarain¹, M. Orszag² and J. Stephany¹

¹ *Departamento de Física, Universidad Simón Bolívar,
Apartado Postal 89000, Caracas 1080A, Venezuela*

² *Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago, Chile*

In this work we show that is possible to obtain Total Quantum Zeno Effect in an unstable systems for times larger than the correlation time of the bath. The effect is observed for some particular systems in which one can chose appropriate observables which frequent measurements freeze the system into the initial state. For a two level system in a squeezed bath one can show that there are two bath dependent observables displaying Total Zeno Effect when the system is initialized in some particular states. We show also that these states are intelligent states of two conjugate observables associated to the electromagnetic fluctuations of the bath.

I. INTRODUCTION

An interesting consequence of the fact that frequent measurements can modify the dynamics of quantum systems is known as Quantum Zeno Effect (QZE) [1, 2, 3, 4, 5]. In general QZE is related to suppression of induced transitions in interacting systems or reduction of the decay rate in unstable systems. Also, the opposite effect of enhancing the decay process by frequent measurements has been predicted and is known as Anti-Zeno Effect (AZE). The experimental observation of QZE in the early days was restricted to oscillating quantum systems [6] but recently, both QZE and AZE were successfully observed in irreversible decaying processes.[7, 8, 9].

Quantum theory of measurements predicts reduction of the decay rate in unstable systems when the time between successive measurements is smaller than the Zeno Time which is known to be smaller than the correlation time of the bath. This effect is universal in the sense that it does not depend on the measured observable whenever the time between measurements is very small. This observation does not preclude the manifestation of Zeno Effect for times larger than the correlation time for some well selected observables in a particular bath. In this work we show that is possible for a two-level system interacting with a squeezed bath to select a couple observables whose measurements beyond the correlation time for adequately prepared systems lead to the total suppression of transitions, i.e Total Zeno Effect.

This work is organized as follows: In section (II) we discuss some general facts and review some results obtained in reference [10] which are needed for our discussion. In Section (III) we define the system we deal with and identify the observables and the corresponding initial states which are shown to display Total Zeno Effect. In section (V) we show that the initial states which show Total Zeno Effect are intelligent spin states, i.e states that saturate the Heisenberg Uncertainty Relation for two fictitious spin operators. Finally, we discuss the results in Section (VI).

II. TOTAL ZENO EFFECT IN UNSTABLE SYSTEMS

Consider a closed system with Hamiltonian H and an observable A with discrete spectrum. If the initial state of the system is the eigenstate $|a_n\rangle$ of A with eigenvalue a_n , the probability of survival in a sequence of S measurements, that is the probability that in all measurements one gets the same result a_n , is

$$P_n(\Delta t, S) = \left(1 - \frac{\Delta t^2}{\hbar^2} \Delta_n^2 H\right)^S \quad (1)$$

where

$$\Delta_n^2 H = \langle a_n | H^2 | a_n \rangle - \langle a_n | H | a_n \rangle^2 \quad (2)$$

and Δt is the time between consecutive measurements. In the limit of continuous monitoring ($S \rightarrow \infty, \Delta t \rightarrow 0$ and $S\Delta t \rightarrow t$), $P_n \rightarrow 1$ and the system is freezed in the initial state.

In an unstable system and for times larger than the correlation time of the bath, the irreversible evolution of the system can be described in terms of the Liouville operator $L\{\rho\}$ by using the master equation;

$$\frac{\partial \rho}{\partial t} = L\{\rho\}. \quad (3)$$

In this case the survival probability in a sequence of S measurements is:

$$P_n(\Delta t, S) = (1 + \Delta t \langle a_n | L\{|a_n\rangle\langle a_n|\} | a_n \rangle)^S \quad (4)$$

Then, the survival probability in the limit of continuous monitoring is time dependent and is easy to show that it is given by

$$P_n(t) = \exp \{ \langle a_n | L\{|a_n\rangle\langle a_n|\} | a_n \rangle t \} \quad (5)$$

In fact for non zero bath correlation time ($\tau_D \neq 0$) one cannot take the continuous monitoring limit and the equation (5) is an approximation since Δt cannot be strictly zero and at the same time be larger than τ_D . In that case this expression is valid only when the time between consecutive measurements is small enough but greater than

the correlation time of the bath. For mathematical simplicity in what follows we consider the zero correlation time limit and then one is allowed to take the limit of continuous monitoring.

From equation (5) one observes that the Total Zeno Effect is possible when

$$\langle a_n | L \{ |a_n\rangle \langle a_n| \} | a_n \rangle = 0 \quad . \quad (6)$$

Then, for times larger than the correlation time, the possibility of having Total Zeno Effect depends on the dynamics of the system (determined by the interaction with the baths), on the observable to be measured and on the particular eigenstate of the observable chosen as the initial state of the system.

If equation (6) is satisfied, then equation (5) must be corrected, taking the next non-zero contribution in the expansion of $\rho(\Delta t)$. In that case the eq. (4) becomes:

$$P_n(\Delta t, s) = \left(1 + \langle a_n | L \{ L \{ |a_n\rangle \langle a_n| \} \} | a_n \rangle \Delta t^2 / 2 \right)^S \quad (7)$$

Then the survival probability for continuous monitoring is

$$P_n(t) = \exp \left\{ \frac{\langle a_n | L \{ L \{ |a_n\rangle \langle a_n| \} \} | a_n \rangle \Delta t}{2} t \right\} \quad (8)$$

In general L is proportional to γ , the decay constant for vacuum. Then as one can see a decay rate proportional to $\gamma^2 \Delta t$ appears. and the decay time is $\propto \frac{1}{\gamma^2 \Delta t}$, which is in general a number much larger than the typical evolution time of the system since $\Delta t \ll \gamma$. This observation is particularly important for system in which one cannot take the zero limit in Δt , i.e when one has a bath with a non zero correlation time. Notice that as the spectrum of the bath gets broader, τ_D becomes smaller, and one is able to choose a smaller Δt , approaching in this way the ideal situation and the Total Zeno Effect.

III. TOTAL ZENO OBSERVABLES

In the interaction picture the Liouville operator for a two level system in a broadband squeezed vacuum has the following structure [11],

$$\begin{aligned} L\{\rho\} = & \frac{1}{2}\gamma(N+1)(2\sigma\rho\sigma^\dagger - \sigma^\dagger\sigma\rho - \rho\sigma^\dagger\sigma) \\ & \frac{1}{2}\gamma N(2\sigma^\dagger\rho\sigma - \sigma\sigma^\dagger\rho - \rho\sigma\sigma^\dagger) \\ & -\gamma M e^{i\phi}\sigma^\dagger\rho\sigma^\dagger - \gamma M e^{-i\phi}\sigma\rho\sigma \end{aligned} \quad (9)$$

where γ is the vacuum decay constant and $N, M = \sqrt{N(N+1)}$ and ψ are the parameters of the squeezed bath. Here σ and σ^\dagger are the ladder operators for a two level system,

$$\sigma = \frac{1}{2}(\sigma_x - i\sigma_y) \quad \sigma^\dagger = \frac{1}{2}(\sigma_x + i\sigma_y) \quad (10)$$

with σ_x, σ_y and σ_z the Pauli matrices.

Let us introduce the Bloch representation of the two level density matrix

$$\rho = \frac{1}{2}(1 + \vec{\rho} \cdot \vec{\sigma}) \quad (11)$$

Using this representation and the master equation one can obtain the following set of differential equation for the components of the Bloch vector (ρ_x, ρ_y, ρ_z) :

$$\begin{aligned} \dot{\rho}_x &= -\gamma(N + 1/2 + M \cos(\psi))\rho_x + \gamma M \sin(\psi)\rho_y \\ \dot{\rho}_y &= -\gamma(N + 1/2 - M \cos(\psi))\rho_y + \gamma M \sin(\psi)\rho_x \\ \dot{\rho}_z &= -\gamma(2N + 1)\rho_z - \gamma \end{aligned} \quad (12)$$

which has the following solutions:

$$\begin{aligned} \rho_x(t) &= (\rho_x(0) \sin^2(\psi/2) + \rho_y(0) \sin(\psi/2) \cos(\psi/2)) \\ &\quad e^{-\gamma(N+1/2-M)t} \\ &\quad + (\rho_x(0) \cos^2(\psi/2) - \rho_y(0) \sin(\psi/2) \cos(\psi/2)) \\ &\quad e^{-\gamma(N+1/2+M)t} \end{aligned} \quad (13)$$

$$\begin{aligned} \rho_y(t) &= (\rho_y(0) \cos^2(\psi/2) + \rho_x(0) \sin(\psi/2) \cos(\psi/2)) \\ &\quad e^{-\gamma(N+1/2-M)t} \\ &\quad + (\rho_y(0) \sin^2(\psi/2) - \rho_x(0) \sin(\psi/2) \cos(\psi/2)) \\ &\quad e^{-\gamma(N+1/2+M)t} \end{aligned} \quad (14)$$

$$\rho_z(t) = \rho_z(0) e^{-\gamma(2N+1)t} + \frac{1}{2N+1} (e^{-\gamma(2N+1)t} - 1) \quad (15)$$

These equations describe the behavior of the system when there are no measurements.

Consider now the hermitian operator σ_μ associated to the fictitious spin component in the direction of the unitary vector $\hat{\mu} = (\cos(\phi) \sin(\theta), \sin(\phi) \sin(\theta), \cos(\theta))$ defined by the angles θ and ϕ ,

$$\sigma_\mu = \vec{\sigma} \cdot \hat{\mu} = \sigma_x \cos(\phi) \sin(\theta) + \sigma_y \sin(\phi) \sin(\theta) + \sigma_z \cos(\theta) \quad (16)$$

The eigenstates of σ_μ are,

$$|+\rangle_\mu = \cos(\theta/2) |+\rangle + \sin(\theta/2) \exp(i\phi) |-\rangle \quad (17)$$

$$|-\rangle_\mu = -\sin(\theta/2) |+\rangle + \cos(\theta/2) \exp(i\phi) |-\rangle \quad (18)$$

If the system is initialized in the state $|+\rangle_\mu$ the survival probability at time t is

$$P_\mu^+(t) = \exp \{ F(\theta, \phi) t \} \quad (19)$$

where

$$F(\theta, \phi) = {}_\mu \langle + | L \{ |+\rangle_\mu {}_\mu \langle + | \} |+\rangle_\mu \quad (20)$$

In this case the function $F(\theta, \phi)$ has the structure

$$F(\theta, \phi) = -\frac{1}{2}\gamma(N+1) \left(\rho_z(0) + \rho_z^2(0) + \frac{1}{2}\rho_x^2(0) + \frac{1}{2}\rho_y^2(0) \right) |+\rangle_{\mu_2} = \sqrt{\frac{N}{N+M}}|+\rangle - i\sqrt{\frac{M}{N+M}}\exp\{-i\frac{\psi}{2}\}|-\rangle \quad (25)$$

$$+\frac{1}{2}\gamma N \left((\rho_z(0) - \rho_z^2(0) - \frac{1}{2}\rho_x^2(0) - \frac{1}{2}\rho_y^2(0)) \right. \\ \left. -\frac{1}{2}\gamma M \rho_x(0)(\cos(\psi)\rho_x(0) - \sin(\psi)\rho_y(0)) \right. \\ \left. +\frac{1}{2}\gamma M \rho_y(0)(\sin(\psi)\rho_x(0) + \cos(\psi)\rho_y(0)) \right) \quad (21)$$

where now $\vec{\rho}(0) = \hat{\mu}$ is a function of the angles..

In figure (1) we show $F(\phi, \theta)$ for $N = 1$ and $\psi = 0$ as function of ϕ and θ . The maxima correspond to $F(\phi, \theta) = 0$. For arbitrary values of N and ψ there are two maxima corresponding to the following angles:

$$\phi_1^M = \frac{\pi - \psi}{2} \quad \text{and} \quad \cos(\theta^M) = -\frac{1}{2(N+M+1/2)} \quad (22)$$

and

$$\phi_2^M = \frac{\pi - \psi}{2} + \pi \quad \text{and} \quad \cos(\theta^M) = -\frac{1}{2(N+M+1/2)} \quad (23)$$

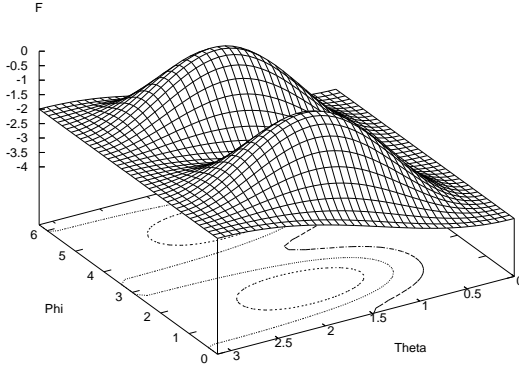


FIG. 1: $F(\phi, \theta)$ for $N = 1$ and $\psi = 0$

These preferential directions given by the vectors $\hat{\mu}_1 = (\cos(\phi_1^M) \sin(\theta^M), \sin(\phi_1^M) \sin(\theta^M), \cos(\theta^M))$ and $\hat{\mu}_2 = (\cos(\phi_2^M) \sin(\theta^M), \sin(\phi_2^M) \sin(\theta^M), \cos(\theta^M))$ define the operators σ_{μ_1} and σ_{μ_2} which show Total Zeno Effect if the initial state of the system is the eigenstate $|+\rangle_{\mu_1}$ or respectively $|+\rangle_{\mu_2}$, then each preferential observable has only one eigenstate displaying Total Zeno Effect. These eigenstates are:

$$|+\rangle_{\mu_1} = \sqrt{\frac{N}{N+M}}|+\rangle + i\sqrt{\frac{M}{N+M}}\exp\{-i\frac{\psi}{2}\}|-\rangle \quad (24)$$

and

The other eigenstates of the observables do not display Total Zeno Effect. As final remark is important to observe that in the previous calculations we have ever chosen the state $|+\rangle_{\mu}$ in order to optimize the function $F(\phi, \theta)$. In fact one can select the state $|-\rangle_{\mu}$ but the final observables displaying Total Zeno Effect will be in the same preferential directions indicated above.

IV. MASTER EQUATION AND MEASUREMENTS

Besides of the Total Zeno effect obtained in the cases specified previously it is also very interesting to discuss the effect of measurements for other choices of the initial state, the states which do not display Total Zeno Effect. To be specific let us consider measurements of the observable $\sigma_{\mu} = \vec{\sigma} \cdot \hat{\mu}$. The modified master equation with the measurement of σ_{μ} is given by [10]:

$$\frac{\partial \rho}{\partial t} = P_{\mu} L\{\rho\} P_{\mu} + (1 - P_{\mu}) L\{\rho\} (1 - P_{\mu}) \quad (26)$$

where

$$P_{\mu} = |+\rangle_{\mu} \langle +| \quad (27)$$

and $L\{\rho\}$ is given by (9). This equation can be solved using the Bloch representation of the density matrix. In this case we can write the density operator in terms of a second set of rotated Pauli matrices that includes the Pauli observable which we are measuring :

$$\rho = \frac{1}{2} (1 + \rho_{\mu} \sigma_{\mu} + \rho_{\alpha} \sigma_{\alpha} + \rho_{\beta} \sigma_{\beta}) \quad (28)$$

where σ_{α} and σ_{β} are two Pauli matrices projected in two orthogonal direction to the vector $\hat{\mu}$. During the process of measurement one obtains always eigenvectors of σ_{μ} observable, these eigenvectors have the property of being zero valued for the other two observables. Then during the measurement process the quantities ρ_{α} and ρ_{β} are equal to zero because these quantities correspond to the mean values of the respective observables. Then in this case the density matrix can be written in term of one parameter which corresponds to the mean value of the observable that is being measured;

$$\rho = \frac{1}{2} (1 + \rho_{\mu} \sigma_{\mu}) \quad (29)$$

with

$$\rho_{\mu} = \langle \sigma_{\mu} \rangle = \text{Tr} \{ \rho \sigma_{\mu} \} \quad (30)$$

Then the master equation is reduced to the following differential equation:

$$\begin{aligned}\dot{\rho}_\mu &= \text{Tr} \{ \dot{\rho} \sigma_\mu \} \\ &= \text{Tr} \{ (P_\mu L \{ \rho \} P_\mu + (1 - P_\mu) L \{ \rho \} (1 - P_\mu)) \sigma_\mu \} \\ &= \text{Tr} \{ L \{ \rho \} \sigma_\mu \}\end{aligned}\quad (31)$$

This equation could induced to think that the evolution with and withouth measurements are equal, but we must remember that the density matrix in the right hand side of (??) is the density with measurements. Substituting the form of the density matrix during the measuring process one can obtain a real differential equation for ρ_μ :

$$\dot{\rho}_\mu = \alpha + \beta \rho_\mu \quad (32)$$

where

$$\alpha = \frac{1}{2} \text{Tr} \{ L \{ 1 \} \sigma_\mu \} \quad (33)$$

$$\beta = \frac{1}{2} \text{Tr} \{ L \{ \sigma_\mu \} \sigma_\mu \} \quad (34)$$

In our case and measuring σ_{μ_1} one obtains

$$\alpha = 2\gamma (N - M + 1/2) \quad (35)$$

and

$$\beta = -\alpha = -2\gamma (N - M + 1/2) \quad (36)$$

The solution to the differential equation is

$$\rho_\mu(t) = 1 + (\rho_\mu(o) - 1) e^{-\alpha t} \quad (37)$$

one can observe the Total Zeno Effect when $\rho_\mu(o) = 1$ which correspond to having as initial state $|+\rangle_{\mu_1}$.

In figure (2) we show the evolution of $\langle \sigma_{\mu_1} \rangle$, that is the mean value of observable σ_{μ_1} , when the system is initialized in the state $|+\rangle_{\mu_1}$. without measurements (master equation (9)) and with frequent monitoring of σ_{μ_1} (master equation (26)). Consistently with our discussion of frequent measurements, the system is freezed in the state $|+\rangle_{\mu_1}$ (Total Zeno Effect).

In figure (3) we show the time evolution of $\langle \sigma_{\mu_1} \rangle$ when the initial state is $|-\rangle_{\mu_1}$ without measurements and with measurements of the same observable as in previous case. One observes that with measurements the system evolves from $|-\rangle_{\mu_1}$ to $|+\rangle_{\mu_1}$. In general for any initial state the system under frequent measurements evolves to $|+\rangle_{\mu_1}$ which is the stationary state of Eq. (26) whenever we do measurements in σ_{μ_1} . Analogous effects are observed if one measures σ_{μ_2} . In contrast, for measurements in other directions different from those defined by $\hat{\mu}_1$ or $\hat{\mu}_2$, the system evolves to states which are not eigenstates of the measured observables.

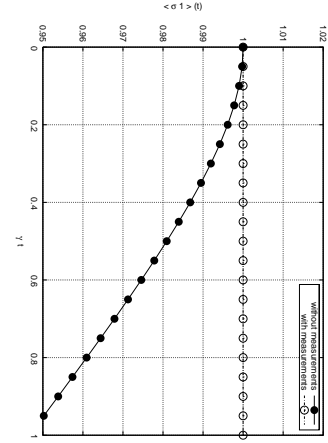


FIG. 2: $\langle \sigma_{\mu_1}(t) \rangle$ for $N = 1$ and $\psi = 0$. Solid circles: no measurements. Empty circles: with measurements. One measures σ_{μ_1} and the initial state is $|+\rangle_{\mu_1}$

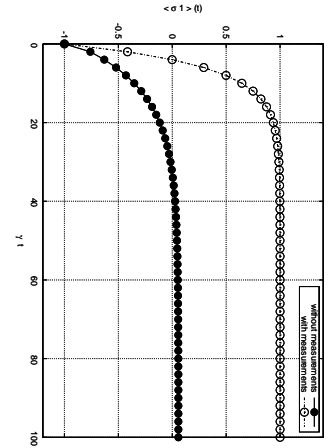


FIG. 3: $\langle \sigma_{\mu_1}(t) \rangle$ for $N = 1$ y $\psi = 0$. Solid circles: no measurements. Empty circles: with measurements. One measures σ_{μ_1} and the initial state is $|-\rangle_{\mu_1}$

V. INTELLIGENT STATES

Aragone et al [12] considered well defined angular momentum states that satisfy the equality $(\Delta J_x \Delta J_y)^2 = \frac{1}{4} | \langle J_z \rangle |^2$ in the uncertainty relation. They are called Intelligent States in the literature. The difference with the coherent or squeezed states, associated to harmonic oscillators, is that these Intelligent States are not Minimum Uncertainty States (MUS), since the uncertainty is a function of the state itself.

In this section we show that the states $|+\rangle_{\mu_1}$ and $|+\rangle_{\mu_2}$ are intelligent states of two observables associated to the bath fluctuations. The master equation (9) can be written in an explicit Lindblad form

$$\frac{\partial \rho}{\partial t} = \frac{\gamma}{2} \{ 2S\rho S^\dagger - \rho S^\dagger S - S^\dagger S \rho \} \quad (38)$$

using only one Lindblad operator S ,

$$S = \sqrt{N+1}\sigma - \sqrt{N}\exp\{i\psi\}\sigma^\dagger \quad (39)$$

$$S = \cosh(r)\sigma - \sinh(r)\exp\{i\psi\}\sigma^\dagger \quad (40)$$

Obviously any eigenstate of S satisfies the condition (6). It is very easy to show that the S operator has two eigenvectors $|\lambda_\pm\rangle$ with eigenvalues $\lambda_\pm = \pm i\sqrt{M}\exp\{i\psi/2\}$. It is also easy to observe that these two states are exactly the same states founded in the previous section, $|\lambda_+\rangle = |+\rangle_{\mu_1}$ and $|\lambda_-\rangle = |+\rangle_{\mu_2}$.

Consider now the standard fictitious angular momentum operators for the two level system are $\{J_x = \sigma_x/2, J_y = \sigma_y/2, J_z = \sigma_z/2\}$ and also two rotated operators J_1 and J_2 which are consistent with the electromagnetic bath fluctuations in phase space (see fig. 2 in ref [10]) and which satisfy the same Heisenberg uncertainty relation that J_x and J_y . They are,

$$\begin{aligned} J_1 &= \exp\{i\psi/2J_z\}J_x\exp\{-i\psi/2J_z\} \\ &= \cos(\psi/2)J_x - \sin(\psi/2)J_y \end{aligned} \quad (41)$$

$$\begin{aligned} J_2 &= \exp\{i\psi/2J_z\}J_y\exp\{-i\psi/2J_z\} \\ &= \sin(\psi/2)J_x + \cos(\psi/2)J_y \end{aligned} \quad (42)$$

These two operators are associated respectively with the major and minor axes of the ellipse which represents the fluctuations of bath.

In terms of J_1 y J_2 we have

$$J_- = \sigma = (J_x - iJ_y) = \exp\{i\psi/2\}(J_1 - iJ_2), \quad (43)$$

$$J_+ = \sigma^\dagger = (J_x + iJ_y) = \exp\{-i\psi/2\}(J_1 + iJ_2). \quad (44)$$

Then S can be written in the following form:

$$S = \exp\{i\psi/2\}(\cosh(r) - \sinh(r))(J_1 - i\alpha J_2) \quad (45)$$

with

$$\alpha = \frac{\cosh(r) + \sinh(r)}{\cosh(r) - \sinh(r)} = \exp\{2r\} \quad (46)$$

Following Rashid *et al* ([13]) we define a non hermitian operator $J_-(\alpha)$

$$J_-(\alpha) = \frac{(J_1 - i\alpha J_2)}{(1 - \alpha^2)^{1/2}} \quad (47)$$

so that

$$S = \exp\{i\psi/2\}(\cosh(r) - \sinh(r))(1 - \alpha^2)^{1/2}J_-(\alpha) \quad (48)$$

After some algebra one obtains that

$$S = 2\lambda_+ J_-(\alpha) \quad (49)$$

From this equation one can observe that the eigenstates of S are also eigenstates of $J_-(\alpha)$ with eigenvalues $\pm 1/2$. It is known that the eigenstates of $J_-(\alpha)$ are intelligent states of J_1 and J_2 , *i.e* they satisfy the equality condition in the Heisenberg uncertainty relation for these observables:

$$\Delta^2 J_1 \Delta^2 J_2 = \frac{|\langle J_z \rangle|^2}{4} \quad (50)$$

VI. DISCUSSION

We have shown that Total Zeno Effect is obtained for two particular observables σ_{μ_1} or σ_{μ_2} , for which the azimuthal phases in the fictitious spin representation depend on the phase of the squeezing parameter of the bath and the polar phases depend on the squeeze amplitude. In this sense, the parameters of the squeezed bath specify some definite atomic directions.

When performing frequent measurements on σ_{μ_1} , starting from the initial state $|+\rangle_{\mu_1}$, the system freezes at the initial state as opposed to the usual decay when no measurements are done. On the other hand, if the system is initially prepared in the state $|-\rangle_{\mu_1}$, the frequent measurements on σ_{μ_1} will makes it evolve from the state $|-\rangle_{\mu_1}$ to $|+\rangle_{\mu_1}$. More generally, when performing the measurements on σ_{μ_1} , any initial state evolves to the same state $|+\rangle_{\mu_1}$ which is the steady state of the master equation (26) in this situation.

The above discussion could appear at a first sight surprising. However, taking a more familiar case of a two-level atom in contact with a thermal bath at zero temperature, if one starts from any initial state, the atom will necessarily decay to the ground state. This is because the time evolution of $\langle \sigma_z \rangle$ is the same with or without measurements of σ_z . In both cases the system goes to the ground state, which is an eigenstate of the measured observable σ_z . In the limit $N, M \rightarrow 0$, $\sigma_{\mu_1} \rightarrow -\sigma_z$, and the state $|+\rangle_{\mu_1} \rightarrow |-\rangle_z$, which agrees with the known results.

Finally, we found that the two eigentates of the two preferential observables displaying QZE are also eigenstates of S operator and consequently intelligent states of J_1, J_2 which are rotated versions of J_x, J_y observables.

A. Acknowledgements

Two of the authors(D.M. and J.S.) were supported by Did-Usb Grant Gid-30 and by Fonacit Grant No G-2001000712.

M.O was supported by Fondecyt # 1051062 and Nucleo Milenio ICM(P02-049)

-
- [1] B.Misra and E.C.G.Sudarshan, J.Math.Phys(N.Y), **18**, 756 (1977)
 - [2] A.Perez and A.Ron, Phys.Rev.A, **42**, 5720(1990)
 - [3] L.S.Schulman, Phys.Rev A, **57**, 1590 (1998)
 - [4] A.D.Panov, Ann.Phys(N.Y), **249**, 5720 (1990)
 - [5] A.G.Kofman and G.Kuritzki, Nature(London), **405**, 546 (2000)
 - [6] W.M.Itano, D.J.Heinzen,J.J.Bollinger and D.Wineland, Phys.Rev.A, **41**, 2295 (1990)
 - [7] S.R.Wilkinson,C.F.Bharucha,M.C.Fischer,K.W.Madison, P.R.Morrow,Q.Miu,B.Sudaram and M.G.Raizen, Nature(London), **387**, 575 (1997)
 - [8] M.C.Fischer,B.Gutierrez-Medina and G.Raizen, Phys.Rev.Lett, **87**, 040402 (2001)
 - [9] P.E.Toschek and C.Wunderlich, Eur.Phys.J.D, **14**, 387 (2001)
 - [10] D. Mundarain and J. Stephany, Phys. Rev. A, **73**, 042113 (2005).
 - [11] C.W.Gardiner, Phys. Rev. Lett., **56**, 1917 (1986).
 - [12] C.Aragone,E.Chalbard and S.Salamo, J.Math.Phys,**17**,1963(1976)
 - [13] M.A.Rashid, J.Math.Phys,**19**,1391(1978)